Physics III
ISI B.Math
Backpaper Exam : December 31,2018

Total Marks: 50
Time : 3 hours
Answer all questions

1. (**Marks:** $8 + 2 = 10$)

(a) In the equilibrium configuration, a spherical conducting shell of inner radius a and outer radius b has a charge q fixed at the centre and a charge density σ uniformly distributed on the outer surface. Find the electric field for all r and the charge on the inner surface. Find the value of the potential at $r = a$. How will your answers to the above questions change if the outer surface is touched with a grounding wire ?

(b) Which one of these is an impossible electrostatic field? (a) $\mathbf{E} = k\frac{\hat{\mathbf{r}}}{r^5}$ (b) $\mathbf{E} = \alpha[y^2\hat{\mathbf{x}} + (2xy + z^2)\hat{\mathbf{y}} + 2yz\hat{\mathbf{z}}]$ where k and α are constants in appropriate units.

2. (**Marks:** $2 + 4 + 4 = 10$)

(a) Why does a *sudden* unplugging of an electrical device like a toaster or an iron often result in drawing a spark?
(b) A battery of emf $\mathcal{E}$ is connected to a circuit of resistance R and inductance L. Find the current in the circuit as a function of time.
(c) Suppose we replace the resistor with a capacitor of capacitance C charged to a potential V and replace the battery by a switch. At time $t = 0$ the switch is closed. Find the current in the circuit as a function of time.

3. (**Marks:** $5 + 5 = 10$)

(a) A steady current I flows down a long cylindrical wire of radius a. Find the magnetic field both inside and outside the wire , if (i) The current is uniformly distributed over the surface of the wire (ii) The current is distributed in such a way that the current density J is proportional to s, the distance from the axis.

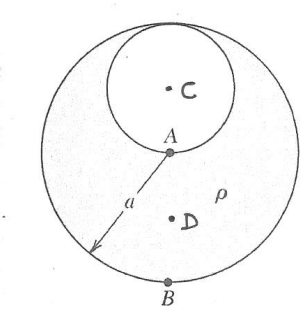
(b) Find the magnetic field at the centre of a square loop that carries a steady current I. Let R be the distance from the centre to any side.

4. (**Marks:** $7 + 3 = 10$)

(a) An electric field which is a solution of source free Maxwell's equations is given by a monochromatic plane wave of amplitude E_0, angular frequency ω, wavelength λ and phase angle zero that is travelling in the negative z direction and polarized in the x direction. Write down the expression for the electric field and the corresponding magnetic field which is also a solution of source free Maxwell's equation. Find the Poynting vector of the electromagnetic field corresponding to this solution.

(b) A point charge q is travelling in the along the x axis at constant speed v. Another point charge q is proceeding at the same speed along the y axis. Argue that mechanical linear momentum is not conserved for this system. No explicit calculation is required, qualitative reasoning considering the direction of the forces should suffice. Does this imply that the law of conservation of momentum does not hold for this system ? Explain.

5. (**Marks:** $5 + 3 + 2 = 10$)



(a) A solid non-conducting sphere of radius a is filled with positive charge of uniform charge density ρ. Then a smaller sphere of radius $\frac{a}{2}$ is carved out as shown in the figure and left empty. What are the direction and magnitude of the electric field $\mathbf{E}$ at A? At B?

(b) Find $\nabla \cdot \mathbf{E}$ and $\nabla \times \mathbf{E}$ at the points C and D , where C and D are points in the interior of the empty region and the charge filled region respectively.

(c) If the non-conducting material was replaced by a conducting material and instead of the uniform charge density ρ(which obviously a conductor cannot sustain) a positive charge q placed at the centre of the cavity at C, Find the electric field at the points D and B.